PRECISION ELECTRO-WEAK AND HADRONIC LUMINOSITY CALCULATIONS

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Abstract

We have used YFS Monte Carlo techniques to obtain per-mil level accuracy for the Bhabha scattering cross section used in the luminosity monitor in electro-weak scattering experiments. We will describe techniques for extending these methods for use in the W production luminosity cross section for hadron colliders.

1 Introduction

Following the discovery of the W and Z bosons, rapid progress was made in precision measurements of electro-weak physics. By the end of LEP's run, high precision Z data reached the 0.1% level, creating pressure for the theoretical calculation of all relevant processes to reach the same level.

The beam luminosity enters into all quantities requiring a normalized cross section, so its precise measurement and calculation are crucial. In e^+e^- scattering, the luminosity is calibrated using small-angle Bhabha scattering, $e^+e^- \to e^+e^- + n\gamma$. This process has both experimental and theoretical advantages: it has a large, clean signal and is almost pure QED, with only a 3% contribution from Z exchange at LEP energies.

The matrix element for small-angle Bhabha scattering was computed by adding the required radiative corrections to reach the desired precision level, and incorporating the resulting matrix element into a Monte Carlo (MC) generator, BHLUMI. [1] The MC algorithm was designed to implement Yennie-Frautschi-Suura (YFS) exponentiation, [2] which rigorously cancels infrared divergences to all orders.

We will review the precision of BHLUMI, and describe the additions to BHLUMI that will be required to go beyond current technology in the event of the construction of proposed e^+e^- linear colliders, which will have larger-angle acceptance for the luminosity monitor. We also describe a proposal for extending the methods developed for precision electro-weak measurements for use in the luminosity monitor for the LHC or other advanced hadron colliders, where W production is a leading candidate for the luminosity process.

2 Two Photon Contributions to the Bhabha Luminosity Process

It was recognized that to reduce the error estimate of BHLUMI to the per-mil level or better, it would be necessary to compute the exact two-photon radiative corrections, which previously had been incorporated in a "leading log" (LL) approximation. The first step was to calculate exactly the cross section for emitting two hard photons. [4] The LL and exact results are compared in Fig. 1(a) for LEP1 parameters (beam energy 91 GeV, angles between 1° and 3°) and LEP2 parameters (beam energy 176 GeV, angles between 3° and 6°). It is seen that the leading log result was accurate to

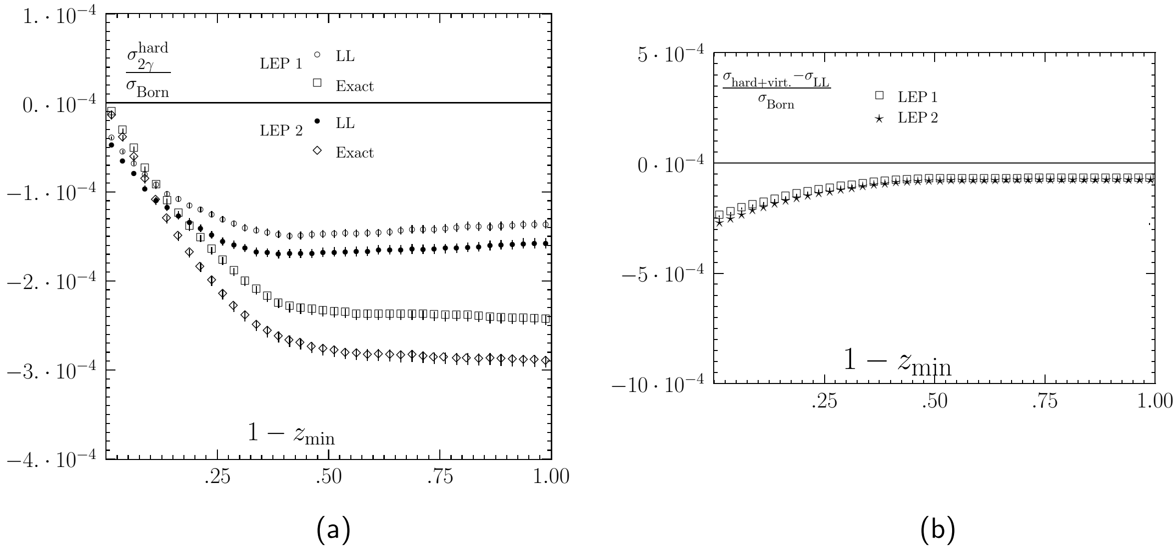

Figure 1: Monte Carlo results (10^6 events) for the $O(\alpha^2)$ cross section for (a) two hard photon emission and (b) single hard plus virtual photon emission from the electron line for LEP1 and LEP2 parameters, where $z_{\rm min}$ is a lower bound on the fraction of the beam energy carried away by the electron and positron. The cross sections are normalized by dividing by the Born cross section, and in (b), the leading log contribution is subtracted.

within 0.013% in both cases.

Subsequently, mixed hard and virtual photon correction to Bhabha scattering were calculated exactly in the small-angle regime. [6] All relevant diagrams were included except for the "box diagrams" shown in Fig. 2, which become significant only at larger angles. The difference between the exact result and the leading log result implemented in BHLUMI is shown in Fig. 1(b) for both LEP1 and LEP2 parameters. In the experimentally interesting range $0.2 \leq 1 - z_{\rm min} \leq 1.0$, BHLUMI is within 0.02% of the exact result for both LEP1 and LEP2.

The second-order photonic corrections were completed by adding the two-loop virtual photon correction to Bhabha scattering from Ref. 7, which yielded a 0.014% contribution to the cross section. [8] The combined contribution of the missing order α^2 photonic radiative corrections in BHLUMI turned out to be 0.027%. The final BHLUMI precision tag was reduced to 0.061% for LEP1 parameters, and to 0.122% for LEP2 parameters.

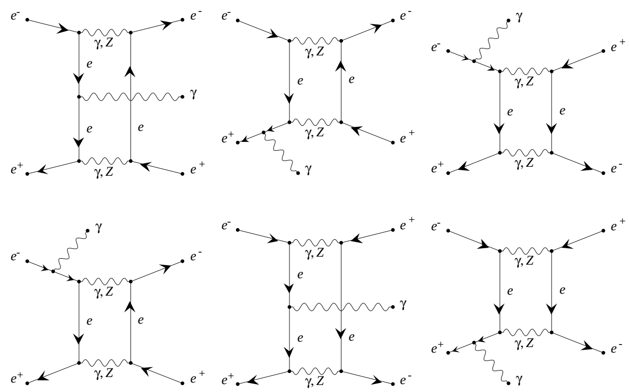

Figure 2: Box diagrams contributing to Bhabha scattering radiative corrections at larger angles. The s channel diagrams are shown. The crossed versions are required for the t channel.

3 Bhabha Luminosity for Linear Colliders

The luminosity monitors for proposed linear e^+e^- colliders will have larger angle acceptances, which requires extending the exact low-angle second order photonic corrections beyond the small-angle regime. The box diagrams in Fig. 2 that were neglected in the previous calculation must be added in this case.

Calculating the box diagrams requires one new ingredient not needed for the previous calculations: a five-point off-shell box diagram. An algorithm for this diagram is currently under construction. When complete, the addition of these box diagrams will complete the exact order α^2 photonic contribution to Bhabha scattering.

4 Hadronic Luminosity Monitor

The proposed W-production luminosity process at the LHC will require at least a 1% precision level for the theoretical contribution to the data analysis. Reducing the theoretical uncertainty to this level will require all first and

second order QCD radiative corrections, as well as first-order electro-weak radiative corrections, and mixed QCD – electro-weak corrections. Due to the large number of graphs, automated techniques are essential. Those displayed in this paper were are excerpted from the output of GRACE. [9]

The first-order electro-weak radiative corrections to $u\overline{d} \to W$ consist of three real photon emission graphs and 19 graphs including a virtual photon or Z. These can be computed with well-known methods. The first order gluonic corrections are likewise known, or can be calculated using well-known techniques.

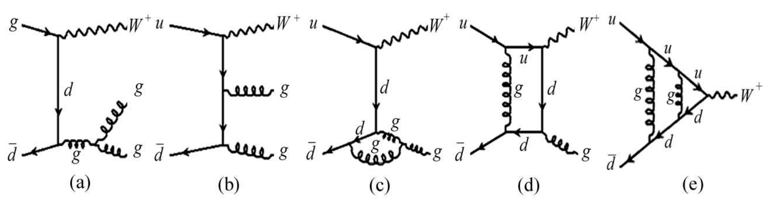

Figure 3: Representative two-gluon diagrams: (a,b) two real gluons, (c,d) real + virtual gluon, (e) two virtual gluons.

The two-gluon radiative corrections are the closest analog to the calculations that were needed in the e^+e^- luminosity case. These are complicated by the triple-gluon coupling in the QCD case, however. Fig. 3 shows some representative examples of the relevant graphs. There are eight graphs for emitting two real gluons, 13 mixed real + virtual gluon (one loop) graphs, and 22 graphs with two virtual gluons (two loop) graphs. The latter clearly present the greatest technical challenges. All of these results will be needed to NLL order to reach the 1% precision level. Thus, they will contribute an $O(\alpha_s^2 L)$ term to the cross section, with L a typical "big logarithm" for the calculation.

The next corrections will be mixed strong and electro-weak radiative corrections, including the representative graphs shown in Fig. 4. There are 10 graphs with a virtual gluon and real photon emission, 86 graphs with a virtual photon or Z and real gluon emission, and 293 two-loop graphs with a virtual gluon and electro-weak loop.

Pure second-order electro-weak radiative corrections will be needed as well, but only to leading log order, adding a $O(\alpha^2 L^2)$ contribution. The matrix elements will be combined with DGLAP evolved structure functions [10] and incorporated into a MC program. Progress on a precision calculation

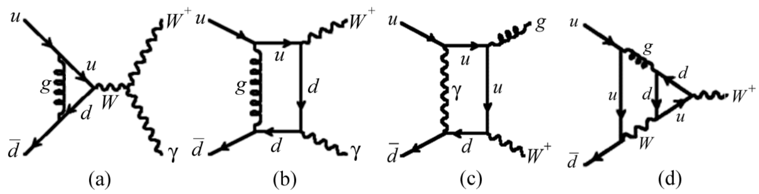

Figure 4: Representative mixed hadronic – electro-weak diagrams: (a,b) real photon + virtual gluon, (c) real gluon + virtual photon, (d) virtual gluon + electro-weak loop.

(0.1%) of the structure function evolution has recently been reported using MC methods. [11]

An important aspect of BHLUMI's success was the YFS exponentiation, which permitted an exact cancelation of all infrared singularities to all orders. We expect YFS exponentiation to play an important role in the hadronic MC as well. Some relevant techniques have already been developed for QCD processes, originally motivated by anticipation of the SSC. [12]

5 Conclusions

We have reviewed the progress which led the electro-weak Bhabha luminosity process to the per-mil precision level and beyond. Verifying this precision required exact calcluations of all second-order photonic radiative corrections to small angle Bhabha scattering. A few "box diagrams," which become important at larger-angle scattering, are still in the process of being calculated. Adding these box diagrams will bring to completion a 12-year project to compute all of these processes.

The construction of the LHC and other next-generation colliders will soon place unprecedented precision requirements on the calculations of the hadronic and electro-weak processes measured at those colliders. A luminosity process calculation on the order of 1% will be needed to fully test the validity of the Standard Model, and to search effectively for hints of new physics.

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References

[1] S. Jadach, W. Płaczek, E. Richter-Wąs, B.F.L. Ward and Z. Wąs, *Comp. Phys. Commun.* **70**, 305 (1992); *Comp. Phys. Commun.* **102**, 229 (1997).

[2] D.R. Yennie, S.C. Frautschi and H. Suura, *Ann. Phys.* **13**, 379 (1961).

[3] S. Jadach, M. Melles, W. Płaczek, E. Richter-Wąs, M. Skrzypek, B.F.L. Ward, Z. Wąs and S.A. Yost, *Tennessee International Symposium on Radiative Corrections*, 153 (World Scientific, Singapore, 1994).

[4] S. Jadach, B.F.L. Ward and S.A. Yost, *Phys. Rev.* **D47**, 2682 (1993).

[5] S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost, *Acta Phys. Polon.* **B30** 1745 (1999).

[6] S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost, *Phys. Lett.* **B377**, 168 (1996).

[7] F.A. Berends, W.L. Van Neerven and G.J.H. Burgers, *Nucl. Phys.* **B297**, 429 (1988).

[8] B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, *Phys. Lett.* **B450**, 262 (1999).

[9] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu and H. Tanaka, KEK report 92-19 (1993); G. Belanger, F. Boudjema, J. Fujimoto, T. Isikawa, T. Kaneko, K. Kato and Y. Shimizu, *LAPTH* preprint LAPH-982/03, KEK preprint KEK-CP-138, hep-ph/0308080 (2003).

[10] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972); L.N. Lipatov, *Sov. J. Nucl. Phys.* **20**, 95 (1975); G. Altarelli and G. Parisi, *Nucl. Phys.* **126**, 298 (1977); Yu. L. Dokshitzer, *Sov. Phys. JETP* **46**, 64 (1977).

[11] S. Jadach and M. Skrzypek, Cracow Inst. Nucl. Phys. HNINP-PAS preprint HNINP-V-04-01, hep-ph/0312355 (2003).

[12] D. DeLaney, S. Jadach, Ch. Shio, G. Siopsis and B.F.L. Ward, *Phys. Rev.* **D47**, 853 (1993); *Phys. Lett.* **B342**, 239 (1995); D. DeLaney, S. Jadach, S.M. Kim, Ch. Shio, G. Siopsis and B.F.L. Ward, *Mod. Phys. Lett.* **A12**, 2425 (1997).